A model for the conceptual learning of mathematics in a technologically enhanced learning environment for first-year prospective mathematics teachers

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Abstract

It is no unknown fact that South African learners, as well as learners world-wide, underachieve in mathematics. Taking into consideration that the quality of mathematics instruction might be one possible factor that has an influence on learners' mathematics achievement, there are valid reasons questioning the conceptual mathematical knowledge of mathematics teachers. In order to facilitate conceptual understanding teachers themselves must possess profound mathematical knowledge. Literature reveals that a technologically enhanced environment can improve the conceptual learning of prospective mathematics teachers. The interwoven nature of attitudes and beliefs towards mathematics on the one hand, and conceptualisation on the other hand make it difficult to discuss the one without the other. Though, due to time constraints, the purpose of this paper is to determine the influence of a dynamic technologically enhanced environment on the conceptualisation of prospective mathematics teachers[1] regarding the function concept. Eventually the author proposes a model for the conceptual learning of mathematics.

Introduction

Over the past few years the conceptual mathematical knowledge of mathematics teachers has become a reason for concern (Hill, Schilling & Ball, 2004:11). The fragmented and insufficient nature of the mathematical knowledge of student teachers and in-service teachers, as facilitators of learning and key figures in the transformation of mathematics teaching and learning (Mapolelo, 1999:715), have serious implications for teacher training. Indications are that maths students who are products of the existing school system are not adequately prepared for the scientific and technological community of the 21st century (Matthee & Roode, 1998:1). In order to facilitate conceptual understanding teachers themselves must possess profound mathematical knowledge. The subject matter knowledge of teachers lack conceptual depth (Bryan, 1999:1) and they are not capable of making conceptual knowledge accessible for learners (Taylor & Vinjevold, 1999:230). One of the reasons offered why learners do not learn

[1] From this point forward prospective teachers are referred to as student teachers or students

mathematics with understanding, is the inadequacy of their teachers' knowledge (Fennema & Franke, 1992:148) that has a negative influence on learners' achievement (Hill, Rowan & Ball, 2005: 371)

Research question

The background as discussed above culminates in the following research question: What is the influence of a technologically enhanced environment on the conceptualisation of student teachers?

Aim of the research

The aim of this research was to determine the influence of a technologically enhanced environment on the conceptualisation of student teachers. The researcher eventually proposes model for the conceptual learning of student teachers in a technologically enhanced environment. Although student teachers' attitudes and beliefs towards mathematics interact with their conceptualisation and therefore also form part of such a model, the focus of this paper is on the conceptualisation of student teachers.

Role of technology

According to the *Principles and Standards* (NCTM, 2000:24) technology is essential in the teaching and learning of mathematics, especially during teacher training, because it influences the meaningful learning of mathematics and enhances the conceptual leaning of student teachers. The availability of technology enables student teachers to focus on higher-order thinking skills such as decision-making, reasoning and problem-solving. The use of dynamic software such as Geometer's Sketchpad® can help them investigate the properties of graphs in a relatively short time (Weaver & Quinn, 1999:84). Geometer's Sketchpad, as a dynamic software program, provides a powerful learning tool for exploration and investigation that can aid student understanding of geometric and algebraic relationships. Dynamic technology engages student teachers in their learning process. Technology used as a tool in the classroom assists students in learning how to learn as well as in learning new mathematical skills (Sherry & Jesse, 2000:3). Interactive technology has the ability to enrich the content of student teachers' learning experiences to provide greater flexibility in the active learning process (Yushau, Mji & Wessels, 2005:19).

Goos *et al*. (2003:10-14) describe four metaphors in which technology can be used in the learning environment. These metaphors guided the researcher in the analysis of the interviews conducted in the empirical study:

Technology as master	Technology as servant	Technology as partner	Technology as extension of the self
Students' knowledge as well as the use of technology are limited to a narrow range of operations. Lack of mathematical under-standing prevents them from evaluating the accuracy of computer answers.	Technology replaces mental or pen and paper calculations in a fast, reliable way, while classroom tasks remain unchanged.	Technology as creative tool increases the power students exercise over their learning, by providing access to new kinds of tasks or new ways of approaching existing tasks.	This is the most sophisticated mode of functioning. It involves users incorporating technological expertise as a natural part of their mathematical and/or pedagogical repertoire.

Mathematical knowledge of student teachers

Mathematical knowledge consists of conceptual, procedural and conditional knowledge (Paris & Cunningham, 1996:134). *Conceptual knowledge* (also known as declarative- or "know that" knowledge) is knowledge rich in relationships and understanding and can be seen as a connected web of knowledge (Ben-Hur, 2006:3; Hiebert & Lefevre, 1986:3-4). By definition, conceptual knowledge cannot be learned by rote, but by thoughtful, reflective mental activity.

Knowing *how* to execute a certain mathematical procedure or *procedural knowledge,* involves the ability to solve mathematical problems through the manipulation of mathematical skills. It includes knowledge of the formal language of mathematics or symbolic representations as well as knowledge of rules, algorithms and procedures (Ben-Hur, 2006:3).

Conditional knowledge emerges as knowing *when* and *why* procedural and conceptual knowledge is applied (Woolfolk, 2007: 258). This form of knowledge assists students in using the applicable procedural and conditional knowledge in problem situations (Schunk, 2000:179). According to Haapasalo (2003: 17), this basically calls upon and considers the following questions: *do I know that, do I know why,* and *do I know how I know*?

Adequate knowledge as well as an awareness of that knowledge determines successful cognitive performance (Lester & Kehle, 2003: 508). This active monitoring and consequent regulating of thought processes is known as metacognition (Flavell,1976:232, Mohamed & Ten Nai, 2005:1). Briefly it can also be viewed as thinking about thinking. It consists of metacognitive knowledge (i.e. the three types of knowledge discussed above) as well as metacognitive control or self-regulation. The latter involves prediction (Lucangeli & Cornoldi, 1997:122), planning, monitoring and evaluation (Ertmer & Newby, 1996:10). The important link between metacognitive knowledge and metacognitive control is reflection.

A Framework for the conceptualisation of functions

The function concept is one of the most fundamental concepts in mathematics. Mathematical problem solving involves a transition from a problem situation to a mathematical representation of that situation. Because functions are the mathematical tools used to describe the relationships among variables we can view these as the core of the mathematical problem solving process (O'Callaghan, 1998:24). The framework for the conceptualisation of functions proposed here, will be grounded in a problem-solving environment and formulated in terms of the use of functions to solve problems.

The framework of O'Callaghan (1998:24-25) consists of four component competencies: modelling, interpreting, translating and reifying, all of which are briefly discussed below:

- Modelling
  The ability to represent a problem situation by means of a mathematical function is known as modelling (Lesh & Lehrer, 2003:109).

The following example illustrates the modelling component:

Ben wants to earn money for his studies by selling hot dogs from a cart in front of a building. He pays the owner of the food cart R35,00 per day for the use of the cart. He sells the hot dogs at R4 each. A hot dog costs R2,50.

(a) How many hot dogs does he have to sell in order to break even?

(b) What will his profit be if he sells 130 hot dogs?

(c) How many hot dogs does he have to sell to make a profit of R100 ?

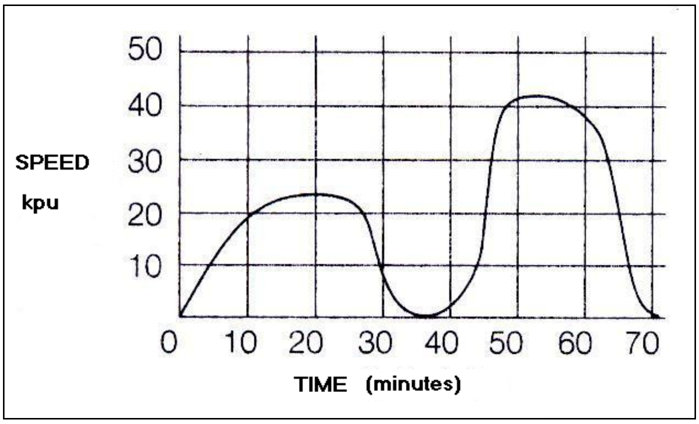
(d) Write down an equation that represents the profit as a function of the number of hot dogs sold.

- Interpretation

The ability to interpret functions in their different representations in terms of real life applications is seen as the reverse process of modelling. The interpretation of graph may require students to think at an abstract level (Van Dyke & White, 2004:116).

Example:

The graph below gives the speed of a cyclist on his daily training ride. The route includes climbing a hill where he pauses for a drink of water- before descending. Use this graph to answer the following questions:



a) Find the speed after 25 minutes.
b) After how many minutes will the speed equal 30 kph?
c) Write down the time intervals when the speed was increasing.
d) How long did it take the cyclist to get to the top of the hill?

- **Translation**

Functions can be represented in different ways, of which the most common are tables, graphs and equations (Van Dyke & Craine, 1997:616), as well as a real life situation (Demana, Schoen & Waits, 1993:13). Translation is the ability to move from one representation of a function to another or to recognise the same function in different representation forms (Demana *et al*. 1993:13) e.g. from the graphic form to an equation, or from table form to the graphic form (O'Callaghan,1998:25). The ability to flexibly move from one form of representation to another, allows students to see rich relationships, develop a better conceptual understanding and enhance their ability to solve problems (Even, 1998:105; Gagatsis & Shiakalli, 2004:654)

The next example illustrates the translation component (O'Callaghan, 1998:30):
Suppose that the following table gives the value (V) in Rand of a car for different numbers of years (t) after it is purchased:

t	V
0	R160 000
2	R135 000
4	R110 000
6	R85 000

Write a formula expressing V as a function of t.

- **Reification**

Reification is defined as the creation of a mental object from what was initially perceived as a process or procedure. The student sees the mathematical object as a single entity that possesses certain properties and it can be operated on by higher level processes such as transformations or composition (O'Callaghan, 1998:25). Reification as the final stage in the learning of the function concept operates as one of the most essential stages in the learning of mathematics (Kieran, 1992:411).

The following example illustrates the reification component :
The number of people living at the coast influences the number of whales in the nearby coastal waters, because the whales don't like the noise of people. Because the whales feed on plankton, the number of whales has an influence on the number of plankton in

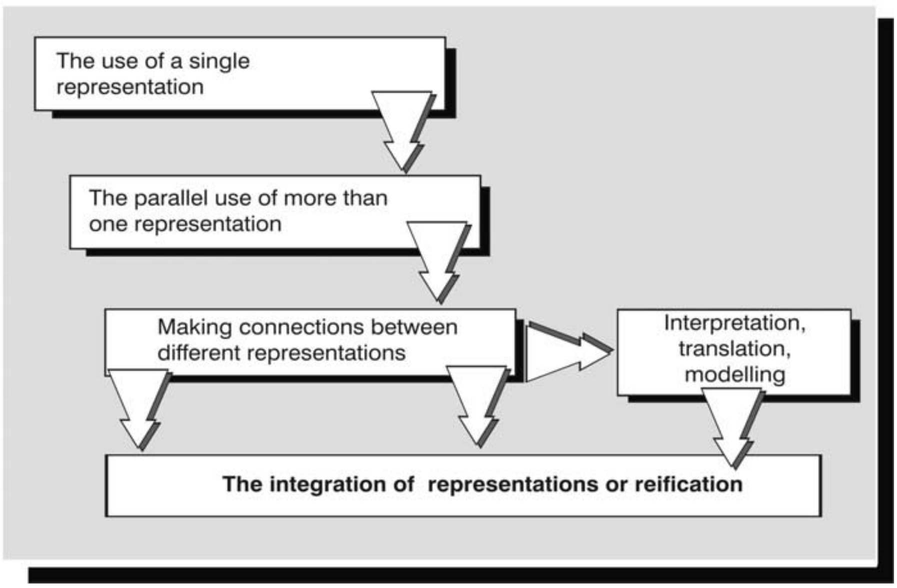
the sea. Suppose the number of people (in thousands) are given by x, and the number of whales by y. A simple model that indicates the relationship between the number of people and the number of whales is given by : $y = f(x) = -\frac{x}{2} + 1000$. If z gives the number of plankton, a simple model is: $z = g(y) = -\frac{y}{5} + 400$.

a) Determine a composite function *h* which gives the number of plankton in terms of the number of people in thousands.
b) Hence determine the number of plankton if the population is 300 000.

Although the framework of O'Callaghan is not a hierarchical model, Dreyfus (1991) proposes that students move through four stages in the learning process, which can be integrated with O'Callaghan's model. The four stages according to Dreyfus (1991:39) are:

- The use of a single (external) representation
- The parallel use of more than one representation
- Making links between parallel representations
- Integrating representations and flexible switching between them.

To come to a better understanding of student teachers' conceptualisation regarding the function concept, an integration of both the models of O'Callaghan and Dreyfus were used in this study.



Research method

In order to answer the research question, I used an explanatory mixed method design. In the quantitative part of the study, two different function tests were administered as a pre- and a post-test to student teachers. In both these tests, conceptualisation of the function concept was measured in terms of the four competence components as discussed above. The participants were a class of 66 student teachers taking mathematic as a major subject. During the intervention program, which forms part of the module, student teachers were introduced to Geometer's Sketchpad as part of the technologically enhanced environment.

In the qualitative part of the investigation, 15 students were then selected form the sample based on their performance in the function pre-test and they were divided in three groups (five high performing, five average performing and five low performing students). Firstly, semi-structured interviews were conducted with the three groups of students to hear their views and experiences regarding the use of technology in mathematics. Afterwards, task-based interviews were individually conducted with these students, in which they had to complete tasks based on the four competence components. In order to analyse the task-based interviews, a model based on Goldin's model (1997:55) has been implemented. Attention has been given to students' use of external representations, formal mathematical notation, the use of metacognitive strategies, as well as affective factors, although only the cognitive and metacognitive factors will be discussed here.

Results

Analysis of the results of the quantitative study revealed that the dynamic technologically enhanced environment did not contribute to an improvement of the student teachers' conceptualisation. Only the reification component showed a practically significant improvement. This is an interesting fact that is also confirmed by Balacheff and Kaput (1996: 469) who found that technology has an influence on the reification of mathematical concepts and relationships.

Analysis of the semi-structured interviews revealed that student teachers had a positive attitude towards the use of technology. From their responses it became clear that technology is a servant or a master to them. To some of them it is still a mechanical execution of procedures without them knowing what they really do.

From the task-based interviews it appears as if students can use one external representation of a function, as long as the questions are based on acquired procedures. It seems as if they are also capable of using more than one parallel representation. As soon as they have to see relationships between different forms of representations, they experience problems. They struggle making interpretations from a graph, most probably because they do not focus on the information in the graph (Mousoulides & Gagatsis, 2004:391). They also appear to have problems translating form the graphical representation form to the algebraic representation form.

Formal mathematical notation also appears to be a problem for most of the students. It seems as if students do not attach the correct meaning to symbolic notation. The inappropriate use of the calculator seemed to prevent them from forming the correct concepts and giving meaning to the correct use of function notation (Greer, 2006: 66).

Analysis of the metacognitive strategies indicated that students overestimated themselves and they do not have the ability to predict with certainty how well they are going to perform in a task. They are also inaccurate in their evaluation of how they did perform in a task after the task has been completed.

Conclusion

It appears as if first year student teachers were not being prepared at school to benefit from the dynamic technologically enhanced environment. One of the questions that was raised by the results of this study is: What can be done in order for student teachers to benefit from a technologically enhanced environment? I propose a model for the effective use of such a learning environment. The model firstly involves a diagnostic assessment of student teachers' basic knowledge of the function concept, their study habits, their attitudes and beliefs with respect to mathematics, as well as their mathematics anxiety. The second component comprises recommendations and support given to student teachers with respect to cognitive and meta-cognitive skills, affective factors and the creation of an advantageous technologically enhanced learning environment.

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